$$\frac{dx^2}{dt} = 2x \frac{dx}{dt}$$

$$\frac{d^2}{dt^2}(x)^2 = 2x\frac{d^2x}{dt} + 2\frac{dx}{dt}\frac{dx}{dt}$$

$$2x\frac{d^2x}{dt} = \frac{d^2}{dt^2}(x^2) - 2\left(\frac{dx}{dt}\right)^2 \quad \text{—— (a)}$$

Dividing both side by 2

$$\frac{\cancel{2}x}{\cancel{2}}\frac{d^2x}{dt} = \frac{1}{2}\frac{d^2}{dt^2}(x^2) - \frac{\cancel{2}}{\cancel{2}}\left(\frac{dx}{dt}\right)^2$$

$$x\frac{d^2x}{dt^2} = \frac{1}{2}\frac{d^2}{dt^2}(x^2) - \left(\frac{dx}{dt}\right)^2 \quad \text{—— (b)}$$

Put eq^n no (4)

$$m\left[\frac{1}{2}\frac{d^2(\bar{x}^2)}{dt^2} - \left(\frac{d\bar{x}}{dt}\right)^2\right] = -\alpha\dot{x}x + F'\bar{x} \quad \text{—— (5)}$$

$$\frac{1}{2}m\frac{d^2}{dt^2}(\bar{x})^2 - m\left(\frac{d\bar{x}}{dt}\right)^2 = -\alpha\dot{x}x + F'\bar{x} \quad \text{—— (6)}$$

$$\cancel{\frac{1}{2}}m\dot{x}^2 = \cancel{\frac{1}{2}}KT$$

$$m\dot{x}^2 = KT$$

$$\boxed{m\left(\frac{dx}{dt}\right)^2 = KT}$$

$$\frac{1}{2}m\frac{d^2}{dt^2}(\bar{x}^2) - KT = -\alpha\bar{\dot{x}}x + 0 \quad \text{—— (7)}$$

$$\boxed{\frac{d}{dt}(\bar{x}^2) = U} \quad \text{—— (c)}$$

$$U = 2x\frac{dx}{dt} = 2x\dot{x}$$

$$\boxed{\dot{x} = \frac{U}{2x}} \text{ ——— (I)}$$

$$\cancel{2}\frac{m}{\cancel{2}}\frac{dU}{dt} - 2KT = -\alpha\frac{U}{2\cancel{\bar{x}}}\cancel{\bar{x}}$$

$$m\frac{dU}{dt} - 2KT = -\alpha U$$

$$\frac{dU}{dt} = -\frac{\alpha}{m}U + \frac{2KT}{m} \text{ ——— (8)}$$

$$\boxed{\frac{du}{dt} + \frac{\alpha}{m}U = \frac{2KT}{m}} \text{ ——— (9)}$$

1st order diff. eqn

$$U = \frac{2KT}{\alpha}$$

$$\left(U = \frac{d(\bar{x})^2}{dt}\right.$$

$$\frac{d(\bar{x}^2)}{dt} = \frac{2KT}{\alpha}dt$$

$$\int d\bar{x}^2 = \int_0^t \frac{2KT}{\alpha}dt \text{ ——— (10)}$$

Integration both side

$$\int d(\bar{x}^2) = \int_0^t \frac{2KT}{\alpha}dt$$

$$\bar{x}^2 = \frac{2KT}{\alpha}t$$

$$(\alpha = 6\pi\eta r)$$

$$\bar{x}^2 = \frac{2KT}{6\pi\eta r}$$

$$\boxed{x^2 = \frac{KT}{3\pi\eta r}\,t}$$